Drivers of the Natural Long-Term Rate of Interest (and Why the Economy Hasn't Tanked)

Martin Ademmer[†] and Jamie Rush[*‡]

[†]*Bloomberg LP, Neue Mainzer Str. 75, 60311 Frankfurt am Main, Germany*

[‡]*Bloomberg LP, 3 Queen Victoria St, London EC4N 4TQ, United Kingdom*

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Abstract

Long-term borrowing costs have been on a path of near continuous decline for the last several decades, with huge consequences for asset values, government finances, borrowers and savers. To shed light on why, we have estimated a global model that brings together an extensive set of drivers of the natural rate of interest for twelve advanced economies, using data spanning half a century.

We find that slower growth and the age structure of the population accounted for about half of the three percentage point decline in the real natural rate for the US between 1970 and a trough in the mid-2010s. Global spillovers account for a bit more than a third of the decline. Our model suggests the natural rate in the US has climbed more than half a percentage point since the trough, partly reflecting extraordinary fiscal stimulus in the US.

Rather than on overnight rates, our model is estimated on ten-year borrowing costs. That is because monetary policy has increasingly relied on squeezing the whole yield curve to support demand. Importantly, when viewed through a longer-term lens, US monetary policy does not look as tight as it does when looking at short-term rates alone. This may help to explain why the economy has been so resilient to the post-pandemic hiking cycle.

JEL classification: C32, E43, G12, J11, O40.

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[*]Correspondence: Jamie Rush, Bloomberg LP, 3 Queen Victoria St, London EC4N 4TQ, United Kingdom. E-mail: jrush36@bloomberg.net.

1 Introduction

Most economists agree that the natural rate of interest – the rate that keeps inflation at target once all transitory shocks have dissipated – has fallen in recent decades. Several drivers have been brought forward to explain the drop, from slower economic growth (e.g. Laubach and Williams (2003)) and demographic shifts (Davis et al. (2023)) to an increased premium for safety and liquidity (Del Negro et al. (2017)) or a higher demand for safe assets (Ferreira and Shousha (2020)). However, there is less agreement about the relative importance of these drivers as well as the role of domestic changes versus global spillovers. And while most studies rely on the policy rate or short-term rate to capture the stance of monetary policy, less is known about the path of long-term rates that is consistent with stable inflation and the drivers of this path.

Our study analyzes the impact of an extensive set of domestic and global drivers on the natural long-term rate of interest for twelve advanced economies deeply enmeshed in the global financial system. To do that, we specify for each economy a model that links the slow-moving trend component of the long-term interest rate to the trends of these drivers. The cyclical component of the long-term interest rate and other macroeconomic variables are assumed to evolve according to a stationary vector autoregressive (VAR) model, similar to Del Negro et al. (2017) and Del Negro et al. (2019). To sharpen up inference of the trends we incorporate survey data on long-run forecasts and external potential growth estimates.

Our study adds to the literature on natural rates in several ways. We estimate our model on long-term borrowing costs rather than on policy rates or short-term rates, in contrast to most other studies. We do so for several reasons: Longer-maturity rates better capture the stance of monetary policy since the global financial crisis, because central banks have increasingly relied on lowering them to support the economy. Some drivers, like the net supply of safe assets and debt issuance, are likely to drive long-term yields directly. And the importance of bonds as a vessel for savings means we should expect the influence of most secular forces to show up in their price (Rogoff et al. (2022)). Finally, ten-year bond yields are crucial benchmarks in global markets.[1]

[1] Several studies, including Pescatori and Turunen (2016), Gonzalez-Astudillo and Laforte (2020) and Zaman (2022), use a 'shadow' short-term interest rate to account for unconventional monetary policies since the global financial crisis. For our purpose, focusing on long-term yields has the additional advantage of having comparable data for all twelve countries considered in our study (for some of which there might be insufficiently rich financial market information to estimate shadow rates).

Moreover, while many studies focus on one or two possible drivers of the natural rate, we round up all the so-called usual suspects to consider an exhaustive set of drivers. They include trend growth and demographic variables but some which don't often feature, such as changes in the relative cost of investment goods, or income inequality. Which is just as well because these appear to be important factors that help determine long-run borrowing costs.

Our model for each economy also captures a broad set of global drivers. These are computed as the weighted average of domestic drivers in other advanced economies that are well-integrated into the global financial system. This allows us to understand how important other countries' variables are to domestic borrowing costs, with the results showing the centrality of US markets to the global determination of interest rates. As shown by Ferreira and Shousha (2020), accounting for global spillovers also sharpens up estimates of the impacts of 'supra-national' drivers such as net safe asset supply and the convenience yield.

Finally, we present detailed results on the drivers of the natural rate for each of the twelve advanced economies in our study. In addition to the US, we consider the UK, Germany, France, Italy, Spain, Sweden, Canada, Australia, New Zealand, Japan and Switzerland.

We identify four findings that advance our understanding of the path of the natural rate, what has been driving it, the role of global factors and the stance of monetary policy:

First, we find that the natural rate is going up again. Our model flags a three percentage point decline in the real natural rate of interest between 1970 and a trough in the mid-2010s – a drop that is similar in magnitude to other estimates in the literature. From there the results show that the natural rate has climbed, mainly because governments have piled on debt since the pandemic, raising the supply of safe assets, increasing competition for savings and lifting fiscal risk.

Second, the most important drivers of the decline in the natural rate of interest in the US have been slower trend growth and shifts in the age structure of the population. After that, cheaper investment goods had the biggest downward influence. The biggest upward influence has been an increase in the net supply of safe assets into global markets.

Third, global factors matter. We find strong evidence that global spillovers play an important role in determining long-term borrowing costs in other countries. For the US global drivers have a one-third weight in determining the natural rate. Reflecting the centrality of the US to the global financial system, for other countries a bigger proportion of local borrowing costs is determined by drivers in other advanced economies.[2]

Fourth, monetary policy has been loose and may not be as tight as it looks when considering short-term interest rates alone. Following the global financial crisis central banks cut borrowing costs to the effective lower bound in many advanced economies. And in several jurisdictions efforts to tackle persistently weak demand in the economy also featured repeated rounds of quantitative easing, driving interest rates lower at the long-end of the yield curve. Our model suggests that monetary policy provided significant support for much of the decade that followed the global financial crisis. Now, with central banks lifting rates to tackle high inflation, our model suggests support has been almost entirely withdrawn. Crucially, though, it does not suggest monetary policy in the US is deep into restrictive territory.

Our results have important policy implications. If pandemic-era fiscal support and post-pandemic government spending has durably tackled persistently weak demand in the economy, we should expect long-term interest rates to remain higher than has been the norm in recent years. Our model suggest ten-year treasury yields in excess of four percent can be expected to endure, as that is where structural determinants of the natural rate suggest it should lie. If that's right, there will be significant fiscal consequences for highly indebted countries. It also has profound implications for asset values, since long-term borrowing costs are highly relevant to the discounting of future cash flows.

With trillions of dollars of assets still parked on central banks' balance sheets, there remains a material dose of monetary support in place, even as short-term interest rates have risen and quantitative tightening has begun. Looking at the stance of monetary policy through a long-term lens shows that, while substantial stimulus has been withdrawn, long-term rates as of the end of 2023 are not yet in deeply restrictive territory. That may be part of the reason why the US economy has not tanked under the strain of the Federal Reserve's hiking cycle.

The remainder of our paper is structured as follows. In the next section we discuss the set of potential drivers that we consider in our model and the connection to previous studies. In Section 3 we explain our modelling strategy and the empirical framework in more detail. In Section 4 we discuss our results, starting with the findings for the US and how they compare to estimates from other studies. We then summarize the main findings for the eleven non-US economies. Section 5 concludes.

[2]The central position of the US in the global financial system and its role for global financial conditions is emphasized, for instance, by Boehm and Kroner (2023) and Miranda-Agrippino and Rey (2020).

2 Potential drivers of the natural rate

We consider an extensive set of potential drivers, drawn from the literature, whose trends are assumed to determine the path of the long-term natural rate. These drivers and the channels through which they might affect the natural rate via their impact on investment or saving behaviour are briefly listed in the following. Exact definitions and data sources are listed in Appendix A, together with the mean and standard deviation of the respective US variables.

Output growth (x_t). The economy's trend growth rate is derived from productivity gains or labor force growth. Both factors might affect the natural rate of interest. For instance, a bigger labor force will need to be equipped with additional capital, raising investment needs. Faster productivity growth tends to raise the return on capital. Laubach and Williams (2003) find a one-for-one relationship between trend output growth and the natural rate. In contrast, Blanchard (2022) and Rogoff et al. (2022) argue against a tight link.

Population age structure (d_t). People likely rely on their parents to support them when they are young, save during their prime working years and spend down their pensions in retirement. If the ratio of dependents to workers goes up, for instance, the supply of savings goes down and the natural rate of interest goes up. This demographic influence is distinct from the impact of aging on the size of the labor force and is emphasized, for instance, by Rachel and Smith (2015) and Davis et al. (2023).

Income inequality (q_t). Higher earners tend to save a bigger proportion of their incomes. So as income inequality goes up, saving is likely to rise, creating downward pressure on the natural rate of interest. Rachel and Smith (2015) conclude that rising inequality within countries might have pushed up global desired saving and created downward pressure on global real rates, even though the overall effect is likely to be significantly smaller than the effect of demographics.

Relative price of investments goods (v_t). As computers and other investment goods have become cheaper, the cost to businesses of upgrading their technology has declined. This might have reduced nominal investment demand and dragged down on the natural rate of interest. Rachel and Smith (2015) argue that the decline in the relative price of investments goods since the 1980s accounts for a substantial part of the fall in real rates. This factor is often overlooked in other studies.

Net safe asset supply (s_t). The worldwide supply of safe assets has climbed in recent decades. At the same time, demand for them, particularly from China, increased in tandem. When demand outstrips supply, the net supply of assets falls pushing down on the natural rate of interest on long-dated bonds. Ferreira and Shousha (2020) find that the net supply of safe assets is a major driver of the natural rate, pushing down during the 2000s and contributing positively since around 2008 as the availability of safe assets surged. This is a supra-national variable that appears in all country-level models (even for those sovereigns not considered safe issuers) to capture spillovers in global debt markets.

Debt issuance (f_t). Beyond the broad balance of safe asset supply, higher domestic indebtedness increases competition for domestic savings and raises local fiscal risks, pushing up on the country-specific premium on bonds and lifting the rate of interest. For some countries this is likely to matter more than others. Moreover, the effect of indebtedness on the natural rate might be non-linear, mattering in particular if the debt-GDP ratio crosses some reference point such as 60% (Ardagna et al. (2007)).

Inflation risk. If uncertainty over the medium-term path of inflation spikes, as it did with the oil price shocks of the 1970s, so does uncertainty over how interest rates might evolve. All that is likely to reduce the attractiveness of locking money away for years at a time, raising the equilibrium interest rate on longer-dated securities. Inflation risk is not directly observable, we proxy it by the trend in the level of inflation (π_t) as higher inflation rates usually go hand in hand with higher inflation volatility (Wright (2011)).

Convenience yield (c_t). Investors are usually willing to accept a lower return for the convenience of holding liquid and safe government bonds. If this premium for safety and liquidity goes up, it will depress the natural rate. Del Negro et al. (2017) provide evidence that the rising premium for the safety and liquidity of Treasury bonds, or the convenience yield, is an important driver of the decline in the natural rate since the mid-1990s. Like Ferreira and Shousha (2020) we include this measure in addition to safe asset demand from foreign governments. That's because it may capture demand created by changes to financial regulation and other investment trends that are not explicitly reflected in other variables in the model. Also like Ferreira and Shousha (2020) the convenience yield enters as a supra-national variable.[3]

Global factors. Because capital flows across borders, developments abroad will have impacts on domestic borrowing costs. We therefore allow for global potential growth (gpx_t),

[3] Since the convenience yield measure is the only driver variable that exhibits notable cyclical fluctuations, we smooth it with a Hodrick-Prescott filter before including it in the model. Imposing instead a relatively low volatility on its trend leads to very similar results (see Section 4.2).

the global population age structure (gd_t), global inequality (gq_t) and the global price of investments (gv_t) to affect the natural rate for the respective domestic economy, with global variables computed as the GDP-weighted sum of the variables for all non-domestic economies. The inclusion of global factors in our model is motivated by the work of Ferreira and Shousha (2020) who emphasize the role of spillovers from drivers of other economies for natural rates. For example, there is no reason to think population aging in Japan would spillover to the age structure of the population in France and so we do not attempt to capture that. Rather the mechanism is that population aging affects spending and saving decisions, and these do spillover to other countries via global capital markets. One other point, we do not model a single global rate, as Cesa-Bianchi et al. (2022) do, because there should be substantial, and interesting variation between countries, especially because we are looking at longer-term borrowing costs.

Our objective is to disentangle the contributions to the change in the natural rate of interest from this wide range of drivers. Strong correlation between the drivers would add to the challenge of doing so (see, e.g. Pescatori and Turunen (2016)). However, for many of our drivers there is little reason to expect that a close systematic relationship is at work. The population age structure, for example, is determined by family planning choices and migration flows. We would not expect those decisions to be closely related to the degree of inequality or the relative price of investment goods. As a diagnostic we tabulate the correlations between the innovations to the trends and find that the links are quite weak (Appendix C).

3 Empirical framework

3.1 Modelling strategy

Our model to estimate the natural rate is similar in spirit to the common trends VAR model of Del Negro et al. (2017) and Del Negro et al. (2019) and is estimated at a quarterly frequency. It rests on three main assumptions to identify the path of the natural rate.

1. *Actual interest rate data tell us something.* Like Del Negro et al. (2017), Del Negro et al. (2019) and others, we assume that borrowing costs in the economy cannot have been too far away from the natural rate on average over the past half century. If they were, then we would have seen spiralling inflation or ever-deepening deflation. In practice the way we take account of this is to impose the restriction that the cyclical component of the

interest rate – i.e. the gap between the observed interest rate and the trend (natural) rate – evolves according to a stationary process.

2. *The natural rate is determined by a set of fundamental economic drivers.* We assume that it is a function of the slow-moving trend components of the drivers listed above. Our model estimates coefficients for the contemporaneous impact of each of the drivers and produces a natural rate path that alongside the other modeling assumptions best fits the actual data observed in markets and the economy.

3. *Economists and market practitioners know something.* We assume that economists' forecasts of where ten-year bond yields will settle once the economic cycle is over (say five years ahead) tell us something about the natural rate. Economists use a lot of tools to think about such questions and so this is a way of shoehorning in wisdom drawn from a holistic set of research methods. As economists' forecasts are subject to error, we place some weight on this anchor but allow plenty of room for fallibility. Long-run expectations from surveys have been exploited in other empirical studies as well to improve interest rate trend estimates or inflation forecasts (e.g. Crump et al. (2018), Del Negro et al. (2017), Chan et al. (2018) and Zaman (2022)).

3.2 Modelling details

Our model consists of three blocks, in line with our three main modelling assumptions. The first block contains a set of core macroeconomic variables, namely the nominal ten-year interest rate (i_t^{10}), inflation rate (π_t) and output growth (x_t), summarized in vector $Y_t^M = (i_t^{10}, \pi_t, x_t)'$. The second block contains all domestic and global drivers discussed in the previous section, besides those that are already part of the first block. These drivers are summarized in vector $Y_t^D = (d_t, q_t, v_t, gpx_t, gd_t, gq_t, gv_t, s_t, f_t, c_t)'$. The third block contains long-run forecasts of the ten-year interest rate (i_t^E), long-run forecasts of inflation (π_t^E) as well as potential growth estimates (px_t). We use these survey data and external model estimates to better identify the latent trends of the core macroeconomic variables in the first block. They are summarized in vector $Y_t^E = (i_t^E, \pi_t^E, px_t)'$.

We decompose Y_t^M into trend components $\bar{Y}_t^M = (\bar{i}_t^{10}, \bar{\pi}_t, \bar{x}_t)'$ and cyclical components $\tilde{Y}_t^M =$

$(\tilde{i}_t^{10}, \tilde{\pi}_t, \tilde{x}_t)'$:

$$Y_t^M = \bar{Y}_t^M + \tilde{Y}_t^M. \tag{1}$$

We model the cyclical components to evolve according to a stationary VAR:

$$\tilde{Y}_t^M = \sum_{p=1}^{P} \Phi_p \tilde{Y}_{t-p}^M + u_t, \quad u_t \sim iid\ N(0, \Sigma_u). \tag{2}$$

The trend components $\bar{Y}_t^M$ are a function of a set of underlying common trends, which we model as random walks. We assume that the trend in the nominal ten-year interest rate equals the trend in inflation plus the trend in the real interest rate, that is

$$\bar{i}_t^{10} = \bar{\pi}_t + \bar{r}_t^{10}. \tag{3}$$

The development of the trend in the real interest rate $\bar{r}_t^{10}$ (or the natural real rate) is the focus of our analysis. We assume it is itself a function of the trend components of the domestic and global drivers. More precisely,

$$\begin{aligned} \bar{r}_t^{10} = &(1-\theta_G)(\theta_x \bar{x}_t + \theta_d \bar{d}_t + \theta_q \bar{q}_t + \theta_v \bar{v}_t) \\ &+ \theta_G(\theta_x g\bar{p}x_t + \theta_d g\bar{d}_t + \theta_q g\bar{q}_t + \theta_v g\bar{v}_t) \\ &+ \theta_s \bar{s}_t + \theta_f \bar{f}_t + \theta_c \bar{c}_t + \theta_\pi \bar{\pi}_t + ot\bar{h}er_t, \end{aligned} \tag{4}$$

with the parameter θ_G reflecting the degree to which $\bar{r}_t^{10}$ is determined by global vs. domestic factors. We further include a 'residual' trend $ot\bar{h}er_t$ in (4) to account for possible omitted factors but only permit very limited volatility.[4]

We identify the trend of the drivers in Y_t^D by linking the observed variables to latent trend components plus some 'cyclical' components, which we simply regard as i.i.d. random measurement errors:

$$Y_t^D = \bar{Y}_t^D + \tilde{Y}_t^D, \tag{5}$$

with $\tilde{Y}_t^D$ given by

$$\tilde{Y}_t^D = \varepsilon_t^D, \quad \text{and } \varepsilon_t \equiv (\varepsilon_t^D, \varepsilon_t^E)' \sim iid\ N(0, \Sigma_\varepsilon). \tag{6}$$

We model the 'cyclical' components $\tilde{Y}_t^D$ as i.i.d. random measurement errors and do not

[4]Including a constant term in (4) instead of a low-volatility trend $ot\bar{h}er_t$ leads to similar results (Section 4.2).

include them in the VAR because most of the drivers in Y_t^D are usually observed only at a lower (e.g. annual) frequency, observations at the beginning of the sample are occasionally missing and because most drivers do not exhibit any obvious cyclical dynamics. At the same time, allowing for measurement errors and considering the trend components in equation (4) instead of linking the observed drivers directly to $\bar{r}_t^{10}$ helps to account for possible noise and outliers in the drivers data.

We improve identification of the latent trends $\bar{Y}_t^M$ by linking them to long-term forecasts from surveys and external potential growth estimates in vector Y_t^E:

$$Y_t^E = \bar{Y}_t^M + \tilde{Y}_t^E, \tag{7}$$

with measurement errors $\tilde{Y}_t^E$ given by

$$\tilde{Y}_t^E = \varepsilon_t^E, \quad \text{and } \varepsilon_t \equiv (\varepsilon_t^D, \varepsilon_t^E)' \sim iid\, N(0, \Sigma_\varepsilon). \tag{8}$$

The long-term forecasts are only available for about the last two thirds of our sample and enter the model at bi-annual and, since 2015, quarterly frequency. Long-term inflation forecasts for the US are available for the full sample.

All underlying common trends of the model can be summarized in vector $\bar{Y}_t = (\bar{\pi}_t, \bar{x}_t, \bar{d}_t, \bar{q}_t, \bar{v}_t, \bar{gpx}_t, \bar{gd}_t, \bar{gq}_t, \bar{gv}_t, \bar{s}_t, \bar{f}_t, \bar{c}_t, \bar{other}_t)'$. As mentioned before, we assume that these evolve according to random walks:

$$\bar{Y}_t = \bar{Y}_{t-1} + e_t, \quad e_t \sim iid\, N(0, \Sigma_e). \tag{9}$$

Writing the model in state-space form, the measurement equations read as

$$\begin{bmatrix} Y_t^M \\ Y_t^D \\ Y_t^E \end{bmatrix} = \Theta \bar{Y}_t + \begin{bmatrix} \tilde{Y}_t^M \\ \tilde{Y}_t^D \\ \tilde{Y}_t^E \end{bmatrix}, \tag{10}$$

where Θ denotes the matrix of impact effects of the common trends $\bar{Y}_t$ on the observed variables. A detailed matrix notation is shown in Appendix B. The transition equations of the state-space model are given by (9), (2), (6) and (8).

We assume that the vector of innovations affecting the trend components e_t, the vector of innovations affecting the cyclical components in the VAR u_t, and the vector of measurement errors ε_t are orthogonal to one another but allow the individual innovations in each vector to be correlated:

$$\begin{bmatrix} e_t \\ u_t \\ \varepsilon_t \end{bmatrix} \sim iid\, N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_e & 0 & 0 \\ 0 & \Sigma_u & 0 \\ 0 & 0 & \Sigma_\varepsilon \end{bmatrix} \right). \tag{11}$$

Orthogonality of innovations to trends and cycles is a common assumption for multivariate unobserved component models (e.g. Banbura and van Vlodrop (2018), Del Negro et al. (2017)) to better disentangle movements in trends from cyclical fluctuations. We also explore potential differences in result when prohibiting cross-section dependence of the elements in equation (7) and (5), i.e. imposing Σ_ε to be diagonal. We find that our results remain almost unchanged (Section 4.2).

3.3 Estimation and prior choices

We estimate the Gaussian state-space model via Bayesian methods over the period from the first quarter of 1968 to the fourth quarter of 2022. For a detailed description of the Gibbs sampling scheme, which uses the simulation smoother of Durbin and Koopman (2002) to obtain draws for the latent states, we refer to the appendix of Del Negro et al. (2017). We use 20,000 draws and discard the first 10,000 draws as burn-in.[5] Note again that most drivers are only available at an annual frequency or, in the case of the inequality measure, not available at all for a good part of the first decade of the sample. Whenever variables are not available for a specific quarter we treat them as missing data, which our procedure can straightforwardly accommodate.

Our prior choices for the impact effects in Θ to be estimated are motivated by recent results from the empirical and theoretical literature. We first discuss our choices for the US model, for which we can draw on the most extensive set of previous studies. Later on, we discuss differences in the prior selection for other countries.

3.3.1 Prior choices for the US

The US prior choices for Θ are summarized in Table 1, along with the posterior estimates discussed in the next section. For the coefficient on trend growth, θ_x, we assume a Gaussian prior

[5] A higher number of draws or using a different seed for the random number generator yields almost identical results.

distribution with mean 0.5 and standard deviation 0.2, which is in the middle ground between the estimated one-for-one relationship in Laubach and Williams (2003) and the arguments in favor of a very weak link emphasized by Blanchard (2022) and Rogoff et al. (2022). Our priors for the coefficients on the population age structure θ_d, income inequality θ_q, and investment prices θ_v are based on the results in Rachel and Smith (2015). We calculated the change in the variable of interest in our sample over the same time period as reported in Rachel and Smith (2015). We then use the reported contribution to the drop in global interest rates to map to our parameter prior. The prior for the coefficient on net safe assets θ_s is drawn from estimates in Ferreira and Shousha (2020) and we assemble the data for that variable in a similar way.

The prior mean of -1 for the coefficient on the convenience yield θ_c follows the model setup in Del Negro et al. (2017). The prior for the debt coefficient θ_f is based on the results in Laubach (2009), which also serve as a rule of thumb in the IMF's debt sustainability analysis (e.g. IMF (2017), Gros and Alcidi (2018)).[6] Large quantitative easing programs around the same time as debt surged could be disguising the impact of debt on yields. We therefore apply a relatively tight prior for this parameter so as not to diminish its importance (Ardagna et al. (2007)). Once central bank balance sheets have returned to more normal sizes, we should expect term premiums on sovereign debt to rise, and there is a rich literature documenting this impact (Kim et al. (2020)). For the coefficient on the inflation risk premium θ_π we lack clear empirical guideline and therefore simply impose a loose prior around 0.5. We assume that the prior distributions for all these coefficients are either truncated to be positive or, in the case of θ_q and θ_c, truncated to be negative. Omitting these restrictions, however, hardly has any impact on our results. Finally, the prior for the global coefficient θ_G follows a Beta distribution with both shape parameters equal to 5, which implies a prior mean of 0.5 and a standard deviation of 0.15. The Beta distribution is a suitable choice for θ_G, as it is defined on the range $[0,1]$. The implied prior mean of 0.5 reflects our desire to remain agnostic about the relative importance of domestic and global factors for the US.

The priors for the variance-covariance matrices Σ_e, Σ_u, and Σ_ε are given by

$$p(\Sigma_e) = IW(\kappa_e, (\kappa_e + n_e + 1)\underline{\Sigma_e}), \tag{12}$$

$$p(\Sigma_u) = IW(\kappa_u, (\kappa_u + n_u + 1)\underline{\Sigma_u}), \tag{13}$$

[6]Rachel and Summers (2019) also argue in favor of a notable link between an increasing debt-GDP ratio and a higher natural rate.

$$p(\Sigma_\varepsilon) = IW(\kappa_\varepsilon, (\kappa_\varepsilon + n_\varepsilon + 1)\underline{\Sigma_\varepsilon}), \tag{14}$$

where $IW(\cdot)$ denotes the inverse Wishart distribution with mode $\underline{\Sigma}$ and κ degrees of freedom, n_e denotes the number of latent trends, and n_u and n_ε denote the number of VAR and non-VAR variables, respectively. The matrices $\underline{\Sigma}$ are assumed to be diagonal.

We set the prior for Σ_e based on the variation of the driver variables in Y_t^D and the variation of the long-term inflation forecasts and potential growth estimates included in Y_t^E. More specifically, we assume that the standard deviation of the expected change in the trends $\bar{Y}_t$ corresponds to the standard deviation of these variables over the sample. As these variables are quite persistent and, apart from measurement errors, closely linked to the underlying trends, this choice results in a still quite conservative prior on the amount of variation in the trends. Our prior choice for the 'residual' trend $\bar{other}_t$, which is not linked to an observed variable, is such that its expected change has a standard deviation equal to 0.5 percentage point over the sample from 1968-2022. Initial conditions for all trend components except $\bar{other}_t$ are based on pre-sample averages. For the initial value of $\bar{other}_t$ we use a loose prior centered around 0.

We set the prior for the variance-covariance matrix Σ_ε of the measurement errors such that the standard deviation of an expected measurement error roughly equals half a standard deviation of the respective observed variable. This rule of thumb allows for bigger measurement errors the more volatile a variable is but still enables us to pin down the trends with some precision. Moreover, the priors for Σ_e and Σ_ε are chosen to be quite tight, with degrees of freedom $\kappa_e = \kappa_\varepsilon = 100$.

The prior for the variance of the VAR innovations Σ_u is an uninformative inverse Wishart distribution centered at a diagonal matrix of 1s, with degrees of freedom $\kappa_u = n_u + 2$. Following Del Negro et al. (2017), we use a standard Minnesota prior for the VAR coefficients Φ but center the prior for the own-lag parameter around zero to describe stationary processes. The hyperparameter for the overall tightness is set to the common value of 0.2 (Giannone et al. (2015)). The VAR model includes $P = 2$ lags. Choosing a lower and higher number of lags for the VAR model ($P = 1$ and $P = 4$, respectively) does not significantly alter our results.

3.3.2 Prior choices for other countries

Estimates of the natural rate for all non-US economies are complicated by the fact that long-term inflation forecasts from surveys are not available in the 1970s and 1980s to help identify the latent inflation trend. At the same time, most of these economies experienced a significantly higher and more volatile inflation rate than the US during this period. We therefore assume that a very persistent trend derived from a simple Hodrick-Prescott filter offers a rough anchor for the inflation trend during the high-inflation periods in the 1970s and 1980s.[7] We then use the information from the Hodrick-Prescott trend for the 1970-1980 period in the same way as we use the information from long-run inflation expectations for more recent decades. We find this anchoring approach results in more credible estimates of the natural rate before 1990 without altering the estimates from 1990 onward in a meaningful way.

We use the US posterior estimates for the coefficients in Θ as prior information for non-US economies. As the US has the most complete data set in our sample and given its position at the centre of the global financial system, we argue that the US estimates for the coefficients in Θ provide a reasonable guide to the strength of the underlying drivers for the other countries. One notable exception is the global coefficient θ_G, which we assume to follow a Beta distribution with shape parameters 15 and 10. This implies a prior mean of 0.6, reflecting our belief that non-US countries are more likely to be buffeted by global drivers.

TABLE 1: Priors and posteriors for the US model

Param.	Prior			Posterior	
	Dist.	Mean	Std.	Median	68% Interval
θ_x	Norm	0.50	0.20	0.640	[0.410, 0.915]
θ_d	Norm	0.05	0.04	0.026	[0.010, 0.047]
θ_q	Norm	-0.15	0.05	-0.079	[-0.129, -0.041]
θ_v	Norm	1.00	0.30	1.002	[0.647, 1.443]
θ_s	Norm	0.15	0.04	0.072	[0.041, 0.105]
θ_f	Norm	0.04	0.01	0.018	[0.007, 0.028]
θ_c	Norm	-1.00	0.50	-0.464	[-0.792, -0.159]
θ_π	Norm	0.50	0.25	0.339	[0.191, 0.492]
θ_G	Beta	0.50	0.15	0.337	[0.319, 0.358]

[7]We use a smoothing parameter $\lambda = 400{,}000$, which leads to a very smooth trend in line with the usual low volatility of long run inflation expectations. As shown later on, this does not prevent us from finding a substantial swing in the inflation trend for most non-US economies in the first half of the sample.

4 Results

4.1 What Drives the US Natural Rate?

We first consider the estimated trend in the nominal rate, plotted in Figure 1 together with the survey long-run expectations that serve as a loose anchor in our model as well as market-implied interest rate expectations. We find that the natural nominal rate of interest for ten-year US government bonds increased from around 8% in 1970 to around 12% in the beginning of the 1980s. It then dropped to a trough of 3.7% in the 2010s. Since around 2020, our results indicate the natural nominal rate has been moving upwards again. As a consequence, the actual ten-year yield remains below the estimated natural nominal rate at least until shortly before the end of the sample period in 2022 Q4. Looking at the long end of the yield curve, monetary policy may therefore not have been as restrictive as increases in short-term policy rates might suggest. This could be part of the reason why the US economy has displayed resilience as the Fed's hiking cycle progressed.

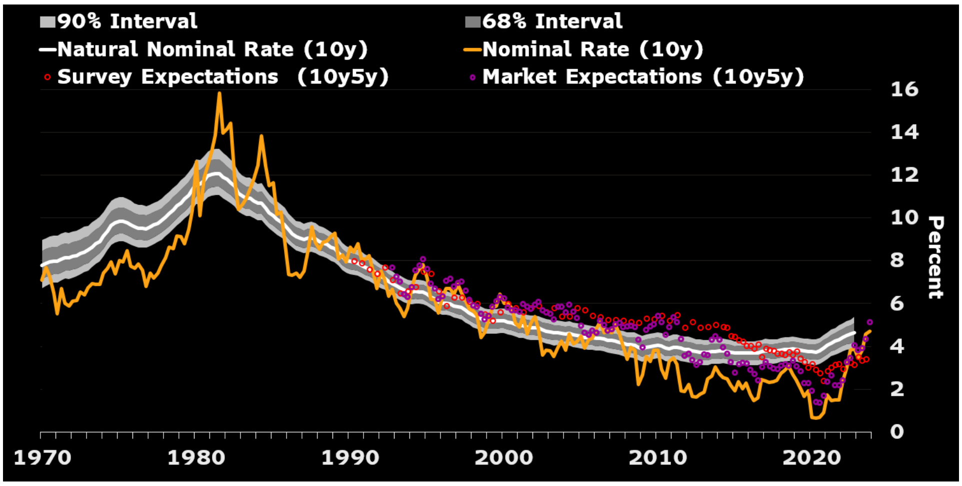

FIGURE 1: The US natural nominal long-term rate 1970-2022

Notes: The white line shows the posterior median of $\bar{i}_t^{10}$, together with 68% and 90% posterior coverage intervals. The yellow line plots the actual nominal ten-year Treasury yield. Red circles indicate long-run expectations of the long-term interest rate from the Consensus Economics survey. Purple circles indicate market-implied interest rate expectations.

Adjusting for trend inflation, we estimate that the natural real rate of interest fell from roughly 5% in the 1970-1980 period to less than 2% in the 2010s (Figure 2). Moreover, we find that not only the natural nominal rate but also the natural real rate has been moving upwards in more recent years. The posterior median estimate rises from its 1.7% low point in the period 2012-2015 to around 2.3% in 2022.

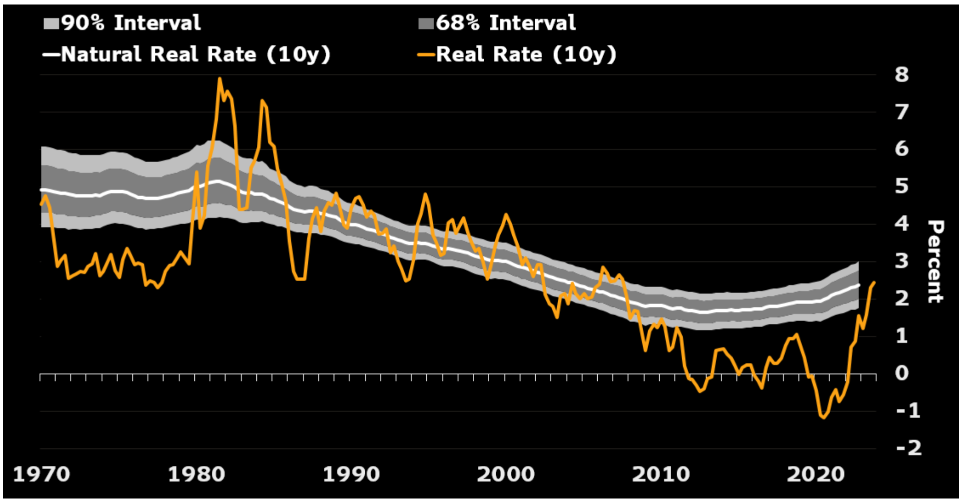

FIGURE 2: The US natural real long-term rate 1970-2022

Notes: The white line shows the posterior median of $\bar{r}_t^{10}$, together with 68% and 90% posterior coverage intervals. The yellow line plots the actual nominal ten-year Treasury yield minus the inflation trend.

What drove the 300 basis point decline in the natural real rate in past decades? And what provoked its rise in more recent years?

Figure 3 shows the contribution of each driver to the change in $\bar{r}_t^{10}$ since 1970. Our results suggest that one of the most important reasons for the drop was weaker trend growth in the US, lowering the natural real rate by almost 1 percentage point. At the beginning of the sample, a swelling workforce and rapid productivity gains meant average annual growth was between 3 to 4%. Strong growth created a powerful incentive to invest – lifting the natural rate. By the 2000s, those drivers were running out of steam. In the wake of the Global Financial Crisis, average annual GDP growth slumped to below 2% (Figure 4). A more sluggish economy meant the attractiveness of investing for the future was weaker – dragging the natural rate lower.

Our results regarding the overall influence of lower trend growth are in the same ballpark as the results of Rachel and Smith (2015), who report a drag of around 100 basis points. With a posterior median of around 0.6, our estimate for the coefficient on trend growth θ_x is very similar to the results of Ferreira and Shousha (2020) but smaller than the estimate of around 1.0 reported in Laubach and Williams (2003).

Shifts in the population age structure constitute another important domestic driver, pushing down $\bar{r}_t^{10}$ by more than 60 basis points. From the 1980s on, as the baby boom generation started squirreling away more funds for retirement, the supply of saving went up – adding more downward pressure on the natural rate. Other domestic drivers contributed to a somewhat

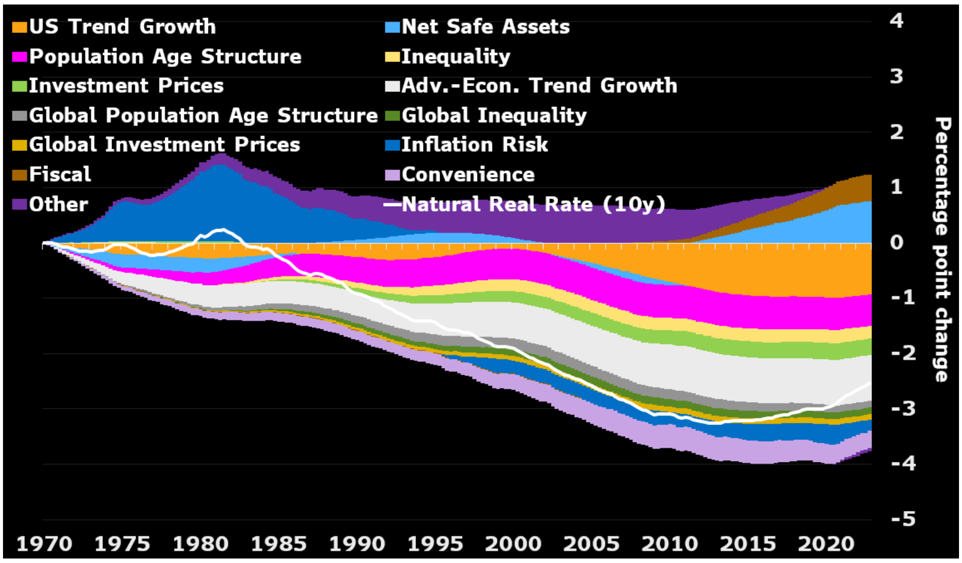

FIGURE 3: Change in the US natural real long-term rate and contributions of individual drivers

Notes: The white line shows the change in the posterior median of $\bar{r}_t^{10}$ since 1970. Contributions to the change in the natural rate are derived from the common trend estimates and their respective impact effects (cf. equation (4)).

lesser extent. We estimate that a decline in the relative price of investment goods lowered the natural rate by about 30 basis points. As computers got cheaper and more powerful, companies did not have to spend so much upgrading their technology. This might have lowered investment demand and ultimately dragged down the natural rate. Rising income inequality in the US might have subtracted another 25 basis points as high earners tuck away a higher share of their income, further increasing the supply of savings. The overall impact of demographic shifts, higher inequality and cheaper investment goods is slightly lower than the numbers in Rachel and Smith (2015) but together their drag on the natural rate turns out to be stronger than that of lower trend growth.

Our results show that a change in global factors ($g\bar{p}x_t$, $g\bar{d}_t$, $g\bar{q}_t$, and $g\bar{v}_t$) is associated with a smaller move in the natural rate than a change in the respective domestic factors, with the posterior median of the global coefficient θ_G coming out at around one-third. However, this still means that global factors played an important role in determining the US natural rate. We find that spillovers ultimately account for more than 120 basis points of the decline in the natural rate in past decades, in particular due to a bigger drop in trend growth in other advanced economies. This drop in foreign trend growth already put significant downward pressure on the US natural rate in the first few decades of the sample period. Weakening US trend growth then added to the downward pressure mostly since around 2000.

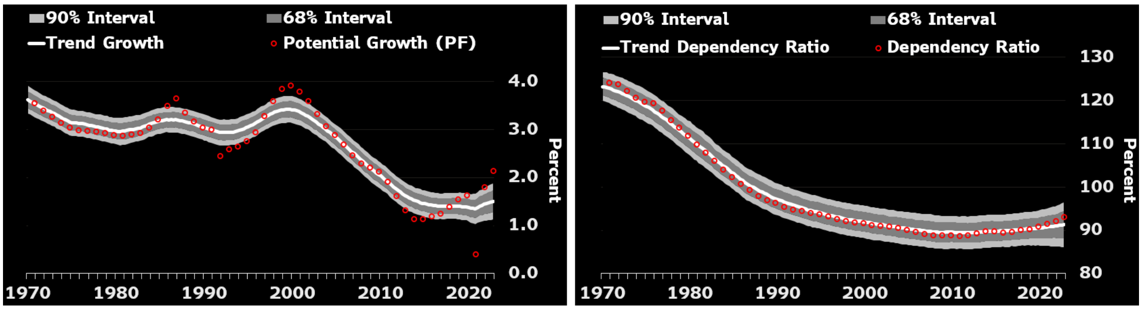

FIGURE 4: Trend components of selected drivers

Notes: Left chart: The white line shows the posterior median of trend growth $\bar{x}_t$, together with 68% and 90% posterior coverage intervals. Red circles indicate annual production function-based estimates of potential GDP growth (px_t). Right chart: The white line shows the posterior median of the population age structure variable $\bar{d}_t$, together with 68% and 90% posterior coverage intervals. Red circles indicate the observed dependency ratio (d_t).

Surging inflation risk raised the natural long-term rate in the 1970s and 1980s, taken by itself. This upward pressure my have been the main reason why overall the natural rate did not decline during this time period despite declining trend growth rates in most advanced economies. The rise in the trend convenience yield helped depress the natural rate, especially from the mid-1990s to around 2015. We estimate that the negative contribution of a higher convenience yield over this period was around 25 basis points, which is broadly similar to the results in Ferreira and Shousha (2020) but below the estimates in Del Negro et al. (2017).

Finally, we find that the more recent rise in the natural real rate can be attributed to two drivers in particular – the net supply of safe assets and US debt issuance. The net supply of safe assets tended to weigh on the natural rate in the first two-thirds of the sample. This was partly due to increasing demand for safe assets from a fast-growing Chinese economy, which saved a lot and channeled those savings into government bonds of the US and other Western economies. In more recent years, however, the stronger increase in the supply of government bonds exerted upward pressure on the natural long-term rate, offsetting the drag from other factors. In the same breath, the soaring indebtedness of the US raised fiscal risk, adding further upward pressure on the natural real rate.

The ‘residual’ trend $\bar{other}_t$ contributes positively to the change in the natural rate throughout the middle of the sample but its impact diminishes in more recent years. As shown in Section 4.2, an alternative model specification that includes a constant term instead does not lead to a material difference in the size of contributions of individual drivers. Such a specification is more restrictive than our baseline specification, as it implicitly assumes that there are no possible time-varying omitted factors.

4.2 Robustness checks

We conduct several robustness checks to assess the sensitivity of our baseline estimates of the US natural rate to different model specifications (Figure 5). We find that, under most, the level of the natural rate of interest and contributions to it are little changed. Shedding the loose anchor provided by survey expectations of long-run interest rates has the biggest impact – this creates space for a bigger drop in trend borrowing costs and bigger increase in recent years.

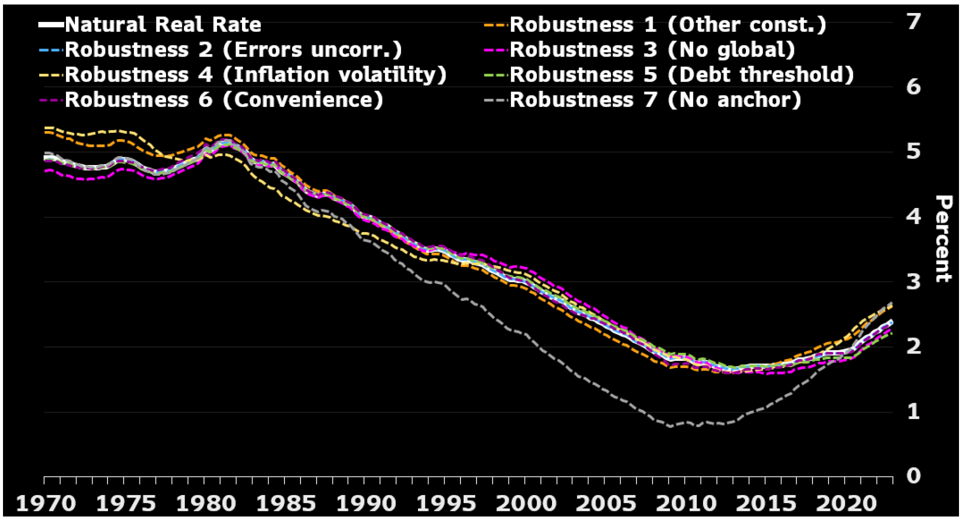

FIGURE 5: The US natural real long-term rate – Robustness checks

Notes: The white line shows the posterior median of $\bar{r}_t^{10}$. Robustness exercise 1: Residual trend $\bar{other}_t$ assumed to be constant. Robustness exercise 2: Variance-covariance matrix Σ_ε assumed to be diagonal. Robustness exercise 3: Global coefficient θ_G assumed to be zero. Robustness exercise 4: External estimate of inflation volatility as proxy for inflation risk. Robustness exercise 5: Lower debt-GDP threshold. Robustness exercise 6: No pre-filtering of convenience yield variable. Robustness exercise 7: Disregarding survey information as anchor.

First, we include a constant term in equation (4) instead of a low-volatility trend $\bar{other}_t$. We find that this leads to slightly higher estimates of $\bar{r}_t^{10}$ at the beginning and the end of the sample and to slightly lower estimates around the 2000-2010 episode. Overall, however, the results are very similar. Moreover, the contributions of the individual drivers to the change in the natural rate remain extremely close to the results of our baseline model (Figure 6).

Second, we explore potential differences when prohibiting cross-section dependence of the elements in equation (7) and (5), i.e. imposing Σ_ε to be diagonal. We find that our results remain almost unchanged, which indicates that the correlation of measurement errors has no visible influence on our estimated effects of the individual drivers.

Third, we banish the global variables gpx_t, gd_t, gq_t and gv_t from our US model by setting $\theta_G = 0$. We find that the overall decline in the natural rate of interest is broadly similar, which

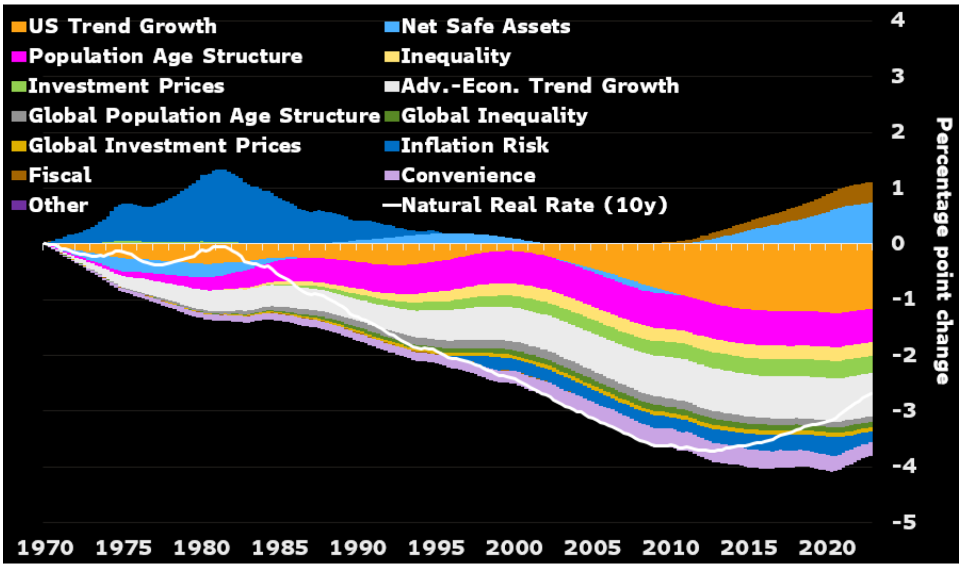

FIGURE 6: Change in the US natural real long-term rate and contributions of individual drivers – Robustness exercise 1

Notes: The white line shows the change in the posterior median of $\bar{r}_t^{10}$ since 1970. Contributions are derived from the common trend estimates and their respective impact effects. Residual trend $\overline{other}_t$ is modelled as a constant term and therefore does not contribute to the change in the natural rate.

underscores the dominant role of the US economy for US yields. The main difference in the contributions arises from the absence of trend growth spillovers from the rest of the world. Given divergent trends in future growth trajectories we think it is useful to account for these influences in the model. For other countries (not illustrated here) removing the global variables would lead to more significant changes in the natural rate estimates, again underlining the importance of the US in global capital markets.

Fourth, instead of using the trend in the level of inflation as proxy for inflation risk, we use an external estimate of inflation volatility based on the model of Chan (2018). The model of Chan (2018) suggests a notable increase in inflation volatility in the period 2020-2022. This contrasts with our baseline proxy, since long-run inflation expectations (which ultimately matter for the trend in the level of inflation) remained relatively flat in recent years. Accordingly, using inflation volatility as alternative proxy results in a slightly stronger rise in the natural real rate at the end of the sample.

Fifth, instead of using the debt-GDP ration in excess of 60% as measure for the driver f_t, we use debt-GDP ration in excess of 30%. Making this change does not lead to a materially different estimate of the natural rate of interest or the contribution of this driver to the change in the natural rate over our sample period.

Sixth, we refrain from smoothing the volatile convenience yield variable with a Hodrick-

Prescott filter before including it in the model but instead impose a relatively low volatility of the convenience yield trend, broadly in line with the volatility of the Hodrick-Prescott trend. We do not find meaningfully different results compared to our baseline.

Finally, seventh, we check what happens if we throw out any information about long-run interest rate expectations from the model. By disregarding the (loose) anchor to economists' long-run expectations, the alternative natural rate estimates follow more closely the observed interest rate path, resulting in a steeper decline in the period 1995-2010 and a stronger rise in more recent years. In line with Zaman (2022), we also find that the link between trend growth and the natural rate is slightly weakened. However, our results do not point to a significant change, with the posterior median of the growth coefficient dropping to 0.55 from 0.64.

4.3 Comparison with estimates from other studies

How do our estimates of the natural rate compare to those from other studies? Since most other studies are concerned with the natural short-term rate, we need to subtract a measure of the term premium trend from our natural long-term rate estimates to allow for a reasonable comparison. We use two different measures of the term premium trend, the Hodrick-Prescott filtered trend of the term premium based on the term structure model of Adrian et al. (2013), adjusted for the estimated impact of large scale asset purchases (Kim et al. (2020)). Alternatively, we use the term premium trend based on the common trends VAR of Del Negro et al. (2017).

We find that our implied natural short-term rate is broadly in the middle of existing estimates at the beginning of the sample and from 2005 onward (Figure 7). It is on the lower end of the spectrum between 1990 and 2005. Our implied natural short-term rate turns out slightly higher when using the term premium trend from Del Negro et al. (2017), which is somewhat lower than our Hodrick-Prescott filtered trend of the term premium.

We conclude from this comparison that our estimated path of $\bar{r}_t^{10}$ is consistent with a broad range of alternative estimates. However, the wide range of results across studies also shows the great uncertainty inherent in models of the natural interest rate.

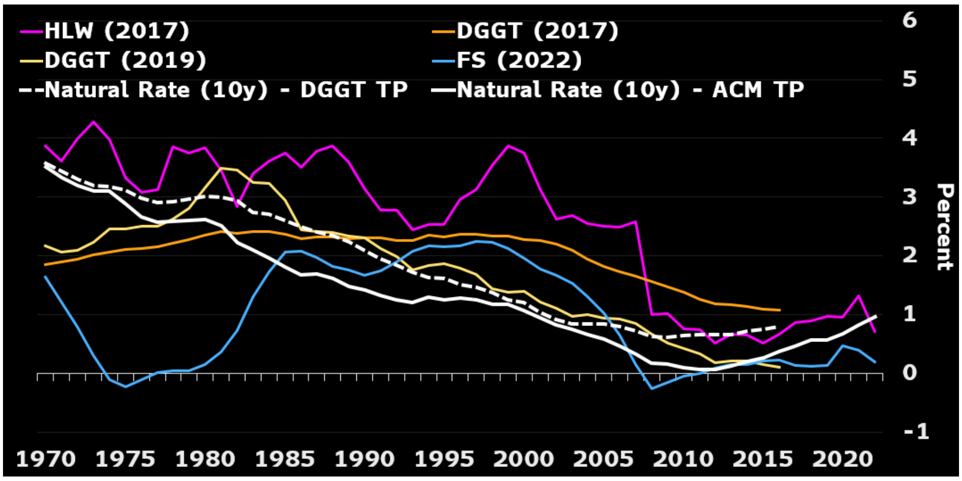

FIGURE 7: Comparison of US results with other studies

Notes: Posterior median of $\bar{r}_t^{10}$ minus the trend of the term premium based on Adrian et al. (2013) (solid white line) or minus the trend of the term premium based on Del Negro et al. (2017) (dashed white line). HLW (2017): Natural short-term rate from Holston et al. (2017). DGGT (2017): Natural short-term rate from Del Negro et al. (2017). DGGT (2019): Natural short-term rate from Del Negro et al. (2019). FS (2022): Natural short-term rate based on Ferreira and Shousha (2020).

4.4 Main findings for other countries

Figure 8 and Figure 9 plot the paths of the natural nominal and real rate, respectively, for all of the twelve advanced economies under study. We find that the natural real rate $\bar{r}_t^{10}$ has moved in similar ways across economies (Figure 9). The variation across economies is larger in the 1970s and 1980s when country-specific risks (in particular inflation risks) varied wildly. It is smaller in the latter part following the great moderation. The US rate is located in the middle range of all estimates and also broadly coincides with a weighted average of all country-specific rates. Our natural rate estimate for Italy is on the upper end of the spectrum for most of the time, our estimates for Japan and Switzerland are on the lower end. Not surprisingly, the variation of the natural nominal rate $\bar{i}_t^{10}$ (Figure 8) across economies is quite pronounced in the 1970s and 1980s due to notable differences in the country-specific inflation trends. We estimate that trend inflation notably exceeds 10% in countries such as Italy and Spain for a good part of the 1970s and 1980s whereas it peaks at slightly below 7% in the US.

Figure 10 shows the contribution of the individual drivers to the change in $\bar{r}_t^{10}$ from 1970 to 2022 for each of the twelve economies. Overall, we find that global drivers play a more important role for all non-US economies than they play for the US, reflecting the dominant position of the US in global capital markets. The posterior median for θ_G varies in the range from 0.5 to 0.75 for non-US economies. The estimates for the UK, Australia and Switzerland are located on the lower end of this spectrum, the estimates for Japan, Germany and Sweden on the upper end.

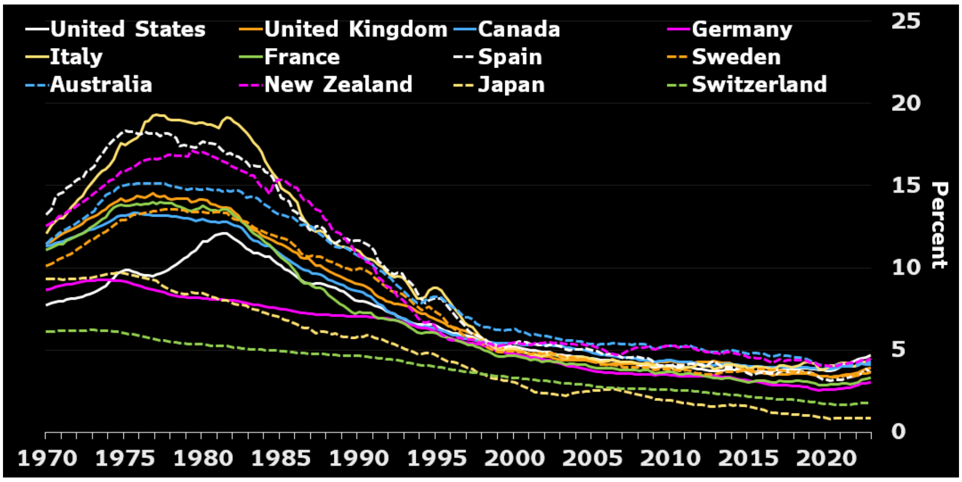

FIGURE 8: Comparison of the natural nominal long-term rate across countries

Notes: The figure plots the posterior median of $\bar{i}_t^{10}$ for each of the twelve advanced economies 1970-2022.

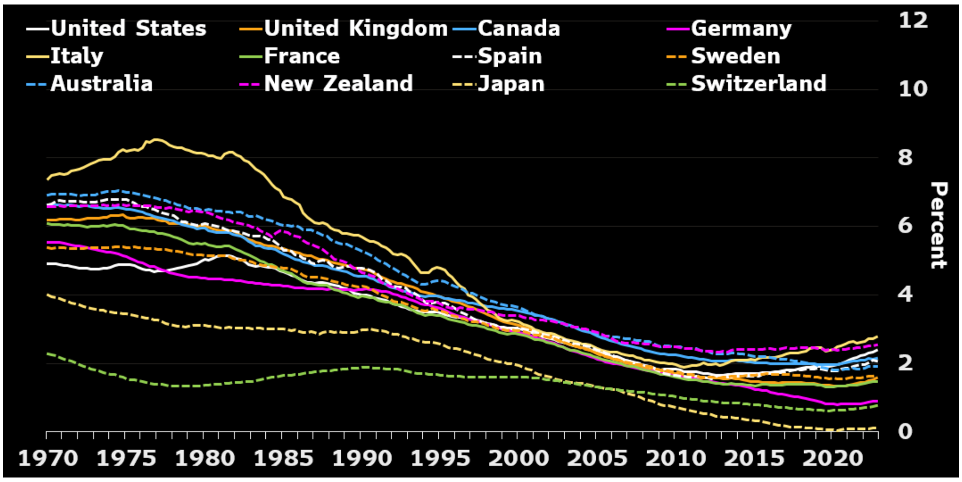

FIGURE 9: Comparison of the natural real long-term rate across countries

Notes: The figure plots the posterior median of $\bar{r}_t^{10}$ for each of the twelve advanced economies 1970-2022.

We find that the main driver of the drop in most countries is slower trend growth, both domestically and among advanced-economy peers. Shifts in the population age structure also dragged, though there are some big differences between countries. For instance, Canada's and New Zealand's prime age dependency ratio fell by almost 40 percentage points between 1970 and 2022, Sweden's and Japan's dependency ratio barely budged. We also find substantial variation in the contributions of domestic fiscal policy and inflation risk, given different levels of indebtedness and experiences during the high inflation period at the start of the sample.

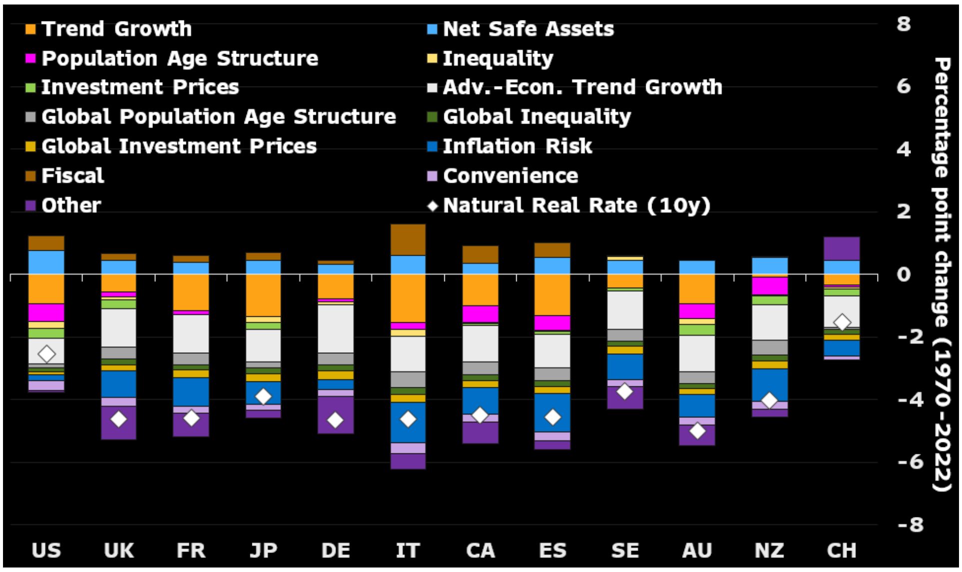

FIGURE 10: Comparison of drivers of the change in the natural real long-term rate across countries

Notes: For each country, the figure plots the change in the posterior median of $\bar{r}_t^{10}$ from 1970 to 2022 and the respective contributions of the individual drivers. Contributions to the change in the natural rate are derived from the common trend estimates and their respective impact effects (cf. equation (4)).

Altogether, our results emphasize that all non-US economies are subject to substantial global spillovers from growth patterns in other advanced economies. Demographic shifts and inflation risk premia play a crucial role for a set of countries but their effects are more heterogeneous across economies. Similar to the US results, the world-wide net supply of safe assets together with country-specific indebtedness provoked upward pressure on the natural rate of most economies, in particular at the end of the sample period.

5 Conclusions

Using a global model consisting of VARs with common trends for twelve advanced economies, we have estimated the impact of twelve drivers of the natural rate of interest using a data set spanning more than half a century. We find that real long-term borrowing costs were pushed down by about three percentage points between 1970 and a trough in the mid 2010s, substantially because trend growth slowed, the population age structure shifted and investment goods became cheaper. Spillovers between the twelve countries are significant, with the results flagging the central importance of the US in global capital markets.

We find that monetary policy has generally been expansionary since the global financial crisis, with policy support having been withdrawn in response to the inflation following the global pandemic. Crucially, policy looks less restrictive viewed through the lens of long-term borrowing costs than when looking at short-term interest rates in isolation. That may help to explain why the US economy displayed so much resilience as the Federal Reserve embarked on the most aggressive rate hiking cycle in a generation.

We estimate that exceptionally loose fiscal policy has pushed up the natural rate in recent years and it would be reasonable to expect long-term borrowing costs to keep rising, should the US government keep running wide deficits. Ten-year Treasury yields climbed well in excess of four percent in late 2023 – our analysis suggests this may become the norm, not the exception.

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A Data

TABLE 2: Variables and data sources

Variable	Definition	Frequency	Source	Mean (US)	Std. (US)
Interest rate i_t^{10}	Ten-year government bond yield	Quarterly	Bloomberg, OECD	6.1	3.1
Inflation rate π_t	Headline inflation, year-on-year	Quarterly	Bloomberg, OECD	3.4	2.2
Output growth x_t	GDP growth, year-on-year	Quarterly	Bloomberg, OECD, St. Louis Fed	2.8	2.3
Population age structure d_t	Dependency ratio (number of people below the age of 22 and above the age of 55, relative to prime-age workers)	Annual	United Nations	99.5	43.6
Income inequality q_t	Proportion of post-tax income going to earners in the top tenth of the distribution (pre-tax data if post-tax unavailable)	Annual	World Inequality Database	28.5	12.3
Relative price of investment goods v_t	Ratio of business investment deflator to the consumption deflator	Annual	National sources	1.2	0.5
Net safe asset supply s_t	Supply of assets from US,UK,DE,FR minus demand for assets, % of world GDP	Annual	Bloomberg, IMF, OECD	14.6	6.7
Debt issuance f_t	Debt-GDP ratio in excess of 60%. Zero, if debt-GDP ratio is below or equal to 60%	Annual	Bloomberg, OECD, national sources	4.1	5.0
Convenience yield c_t	Baa corporate bond yield minus US Treasury yield, Hodrick-Prescott trend	Quarterly	Bloomberg	2.2	0.3
Global variables gpx_t, gd_t, gq_t, gv_t	GDP-weighted sum of respective variable of all other countries			-	-
Long-run forecast interest rate i_t^E	Median long-run forecast of professional forecasters	Bi-annual since 1990, quarterly since 2015	Consensus Economics	5.0	2.6
Long-run forecast inflation rate pi_t^E	Median long-run forecast of professional forecasters. US: Long-run forecast from FRB/US	Bi-annual since 1990, quarterly since 2015. US: Quarterly since 1968	Consensus Economics, Federal Reserve Board	3.3	1.5
Potential growth estimate px_t	Production function-based estimate of potential GDP growth	Annual	Bloomberg, OECD, Penn World	2.7	1.2

B Measurement equation in matrix notation

In more detail, the measurement equation (10) reads as

$$\begin{bmatrix} i_t^{10} \\ \pi_t \\ x_t \\ d_t \\ q_t \\ v_t \\ gpx_t \\ gd_t \\ gq_t \\ gv_t \\ s_t \\ f_t \\ c_t \\ i_t^E \\ \pi_t^E \\ px_t \end{bmatrix} = \begin{bmatrix} (1+\theta_\pi) & (1-\theta_G)\theta_x & (1-\theta_G)\theta_d & (1-\theta_G)\theta_q & (1-\theta_G)\theta_v & \theta_G\theta_x & \theta_G\theta_d & \theta_G\theta_q & \theta_G\theta_v & \theta_s & \theta_f & \theta_c & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ (1+\theta_\pi) & (1-\theta_G)\theta_x & (1-\theta_G)\theta_d & (1-\theta_G)\theta_q & (1-\theta_G)\theta_v & \theta_G\theta_x & \theta_G\theta_d & \theta_G\theta_q & \theta_G\theta_v & \theta_s & \theta_f & \theta_c & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\pi}_t \\ \bar{x}_t \\ \bar{d}_t \\ \bar{q}_t \\ \bar{v}_t \\ g\bar{p}x_t \\ \bar{gd}_t \\ \bar{gq}_t \\ \bar{gv}_t \\ \bar{s}_t \\ \bar{f}_t \\ \bar{c}_t \\ ot\bar{h}er_t \end{bmatrix} + \begin{bmatrix} \tilde{i}_t^{10} \\ \tilde{\pi}_t \\ \tilde{x}_t \\ \tilde{d}_t \\ \tilde{q}_t \\ \tilde{v}_t \\ g\tilde{p}x_t \\ \tilde{gd}_t \\ \tilde{gq}_t \\ \tilde{gv}_t \\ \tilde{s}_t \\ \tilde{f}_t \\ \tilde{c}_t \\ \tilde{i}_t^E \\ \tilde{\pi}_t^E \\ \tilde{px}_t \end{bmatrix}. \tag{15}$$

C Further estimation results (US model)

TABLE 3: Posteriors for the variance-covariance matrix of the innovations to the trends

Innovation	Prior variance (Diag. elem. of $\underline{\Sigma_e}$)	Posterior variance Median	68% Interval
e_t^{π}	1.02e-02	1.70e-02	[1.50e-02, 1.93e-02]
e_t^{x}	2.91e-03	4.49e-03	[3.86e-03, 5.25e-03]
e_t^{d}	6.55e-01	8.70e-01	[7.54e-01, 1.00e+00]
e_t^{q}	1.82e-02	2.42e-02	[2.10e-02, 2.80e-02]
e_t^{v}	1.82e-04	2.34e-04	[2.04e-04, 2.70e-04]
e_t^{gpx}	6.55e-03	8.97e-03	[7.82e-03, 1.04e-02]
e_t^{gd}	3.06e-01	4.17e-01	[3.63e-01, 4.79e-01]
e_t^{gq}	1.64e-02	2.31e-02	[2.00e-02, 2.69e-02]
e_t^{gv}	4.55e-05	6.25e-05	[5.41e-05, 7.27e-05]
e_t^{s}	9.20e-02	1.55e-01	[1.33e-01, 1.82e-01]
e_t^{f}	4.02e-01	6.07e-01	[5.23e-01, 7.09e-01]
e_t^{c}	4.09e-04	3.75e-04	[3.38e-04, 4.19e-04]
e_t^{other}	1.14e-03	1.61e-03	[1.36e-03, 1.90e-03]

TABLE 4: Correlation structure of the innovations to the trends

	e_t^{π}	e_t^{x}	e_t^{d}	e_t^{q}	e_t^{v}	e_t^{gpx}	e_t^{gd}	e_t^{gq}	e_t^{gv}	e_t^{s}	e_t^{f}	e_t^{c}	e_t^{other}
e_t^{π}	1.00	-0.09	0.00	-0.12	0.12	-0.09	0.04	-0.12	0.08	-0.14	0.00	0.03	-0.02
e_t^{x}		1.00	0.02	0.03	0.01	0.07	-0.03	0.02	0.01	0.01	-0.07	-0.06	0.04
e_t^{d}			1.00	-0.06	0.04	0.08	0.10	-0.08	0.09	-0.01	0.01	-0.07	-0.05
e_t^{q}				1.00	-0.08	-0.04	-0.08	0.09	-0.09	0.06	0.00	0.04	0.04
e_t^{v}					1.00	0.02	0.05	-0.08	0.07	-0.07	-0.03	-0.03	-0.01
e_t^{gpx}						1.00	0.06	-0.04	0.06	0.03	-0.02	-0.09	-0.03
e_t^{gd}							1.00	-0.11	0.10	0.01	0.07	-0.07	-0.08
e_t^{gq}								1.00	-0.10	0.00	-0.02	0.07	0.06
e_t^{gv}									1.00	-0.03	0.00	-0.06	-0.04
e_t^{s}										1.00	0.19	-0.10	-0.09
e_t^{f}											1.00	-0.05	-0.10
e_t^{c}												1.00	0.05
e_t^{other}													1.00

Notes: Values refer to posterior median estimates.